Self Supervised Learning Methods for Imaging

Part 1: Introduction

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Tutorial Schedule

PART I: Introduction to Imaging Inverse Problems

Inverse problem framework; ill-posed problems; dimensionality; noise; deep learning solutions; supervised versus unsupervised; learning vs inductive bias.

PART II: Unsupervised methods for Invertible Forward Operator

Blind Denoising; SURE; Noise2X; denoising autoencoders; measurement splitting; GSURE.

PART III: Learning from multiple operators

The impossibility of learning from an incomplete measurement operator; learning from multiple operators; Noise2Noise revisited; measurement splitting revisited; multi-operator consistency; handling noise.

PART IV: Unsupervised methods for ill-conditioned inverse Problems with a single operator

Exploiting Invariance and symmetries; equivariant and non-equivariant operators; enforcing equivariance; handling noise

PART V: Identifiability Theory

Identifiability and dimension; learning with noise; learning from incomplete measurements; Cramer-Wold theorem; generic identifiability

PART VI: Summary and Future Perspectives

The Inverse problem

Goal: estimate signal $\boldsymbol{x}$ from $\boldsymbol{y}$

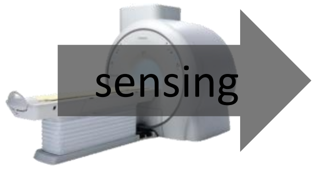
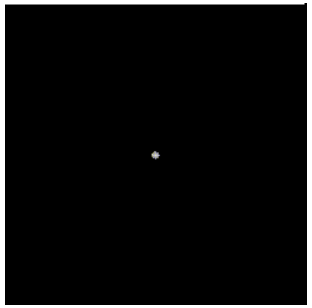
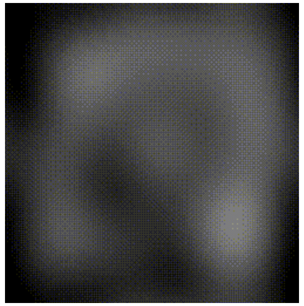
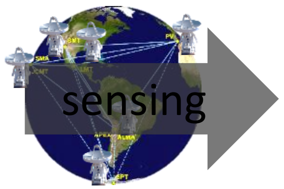
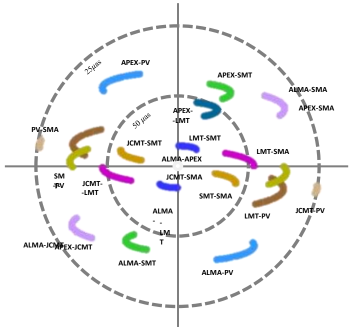
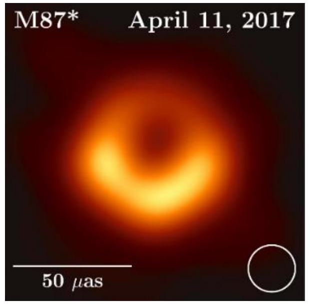
signal $\in \mathbb{R}^n$

$$\underset{\text{measurements } \in \mathbb{R}^m}{} \; \boldsymbol{y} = A(\boldsymbol{x}) + \boldsymbol{\epsilon} \; \text{noise/error}$$

Physics

We will focus on linear problems where the forward operator A is a matrix

Examples

	x	A	y		reconstruction	
Magnetic Resonance Imaging (MRI) A: undersampled Fourier models		 sensing		recover		Source: Brian Hargreaves
Black Hole Imaging A: spatial-frequency e.g. Event Horizon Telescope (EHT)	?	 sensing		recover		The Astrophysical Journal Letters, vol. 875, no. L1, 2019.
Cryogenic electron microscopy (Cryo-EM) A: 2D projections of protein particles		 sensing		recover		Covid-19 virus' structure D. Wrapp et al. *Science*, vol. 367, no. 6483, 2020.

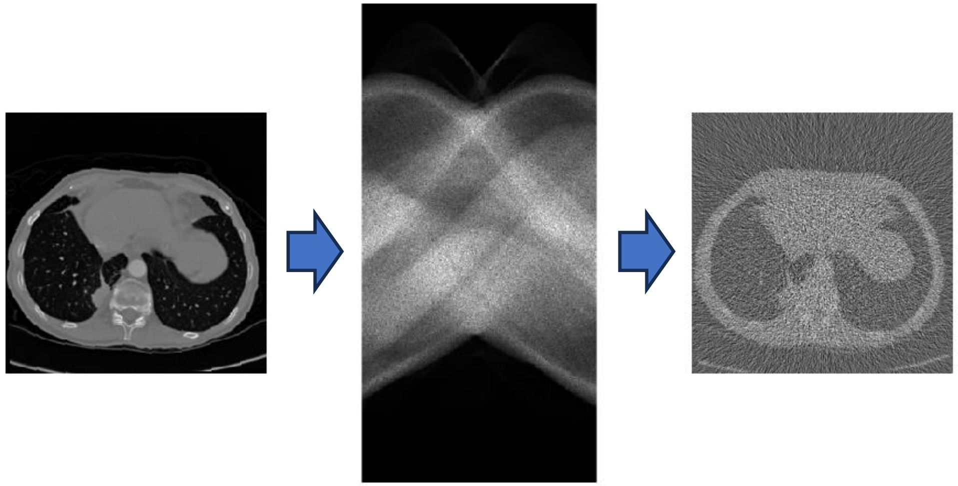
Why it is hard to invert?

Measurements are usually corrupted by noise, e.g.

$$\boldsymbol{y} = A\boldsymbol{x} + \boldsymbol{\epsilon}$$

Can be additive, as above, or more complex, e.g. Poisson.

- Often, we do not know the exact noise distribution
- The forward operator may be poorly conditioned



with noise

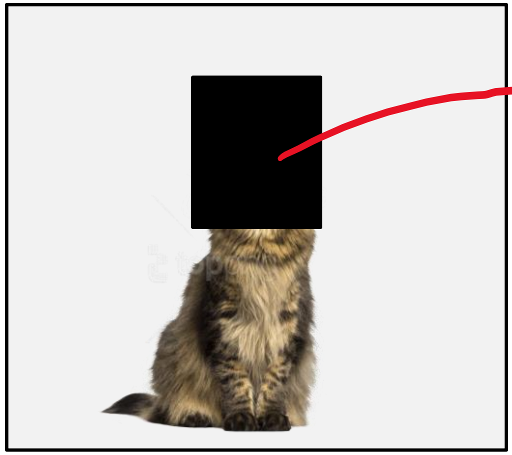
Why it is hard to invert?

Even in the absence of noise, A may not be invertible, giving infinitely many $\hat{\boldsymbol{x}}$ consistent with $\boldsymbol{y}$:

$$\hat{\boldsymbol{x}} = A^{\dagger}\boldsymbol{y} + \boldsymbol{v}$$

where $A^{\dagger}$ is the pseudo-inverse of A and $\boldsymbol{v}$ is any vector in nullspace of A

Unique solution only possible if set of signals $\mathcal{X}$ is low-dimensional



NULLSPACE OF A

reconstruct

Low Dimensional Signal Models

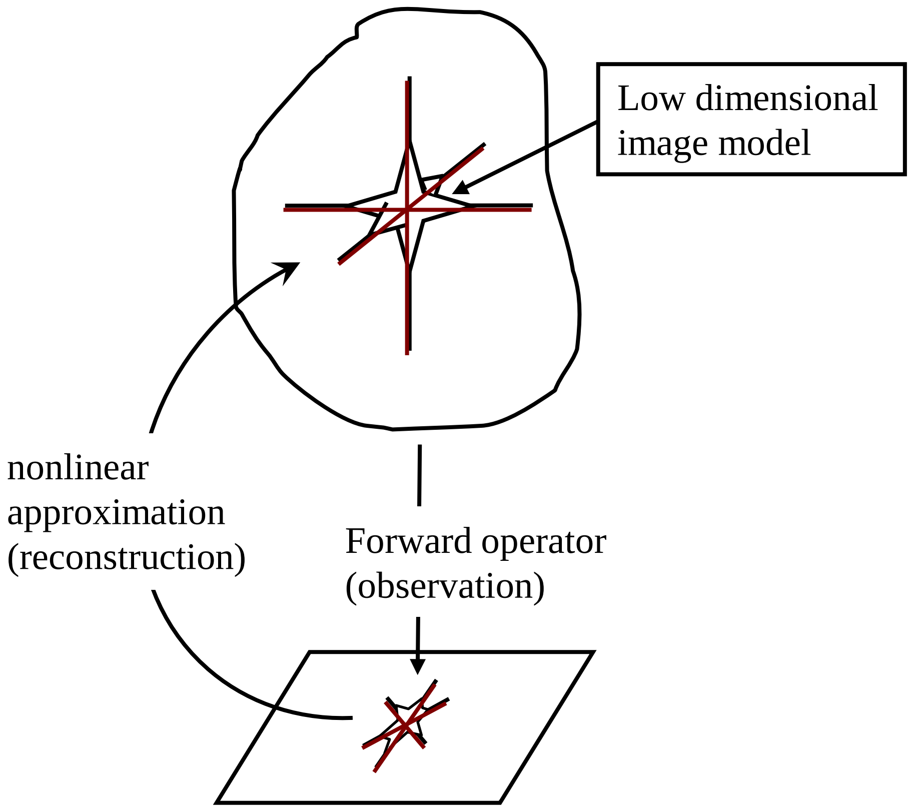
Idea: assume approximate low dimensional image model:

$$\dim \mathcal{X} = k \ll n$$

Examples: sparsity, low-rank, manifolds

Signal Embedding: if $\mathrm{m} \geq \mathcal{O}(k)$ then the problem is approximately one-to-one and (nonlinearly) invertible

This is the principle behind compressed sensing, but is implicit in most inverse imaging problems



Regularised reconstruction

Idea: define a loss $\rho(\boldsymbol{x})$ that promotes plausible reconstructions

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \; ||\boldsymbol{y} - A\boldsymbol{x}||^2 + \rho(\boldsymbol{x})$$

Examples: total-variation, sparsity, etc.

Disadvantages: hard to define a good $\rho(\boldsymbol{x})$ in real world problems, loose with respect to the true signal distribution

Learning approach

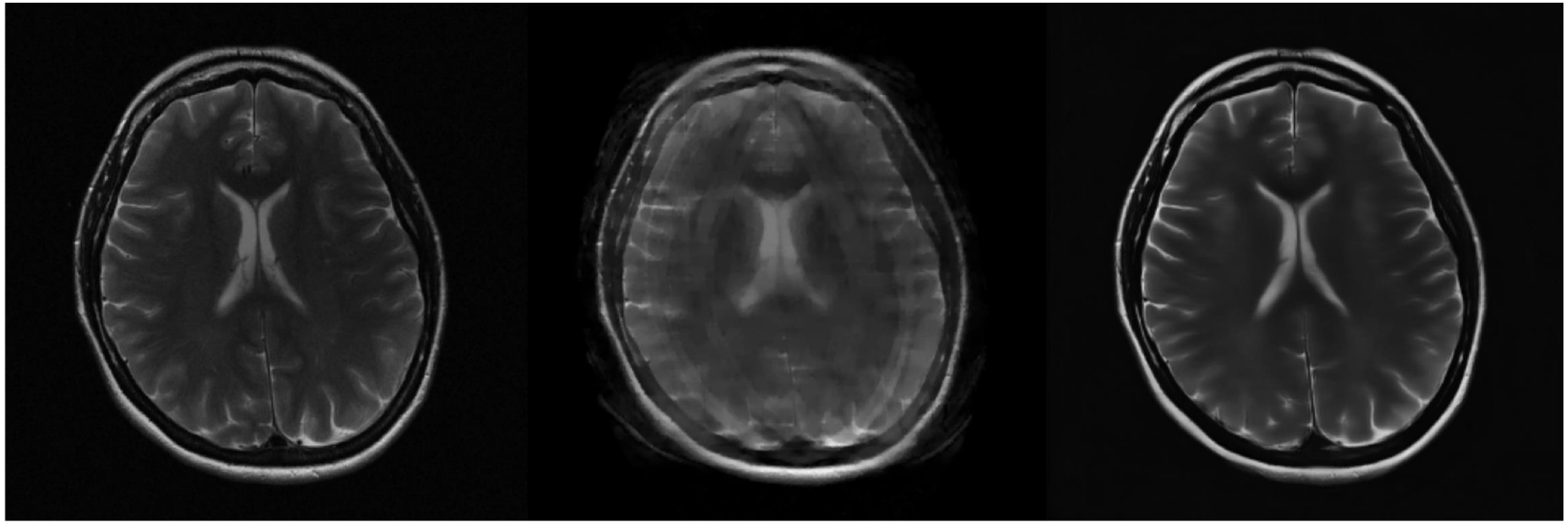
Advantages:

- State-of-the-art reconstructions
- Once trained, f_θ is easy to evaluate

fastMRI

Accelerating MR Imaging with AI

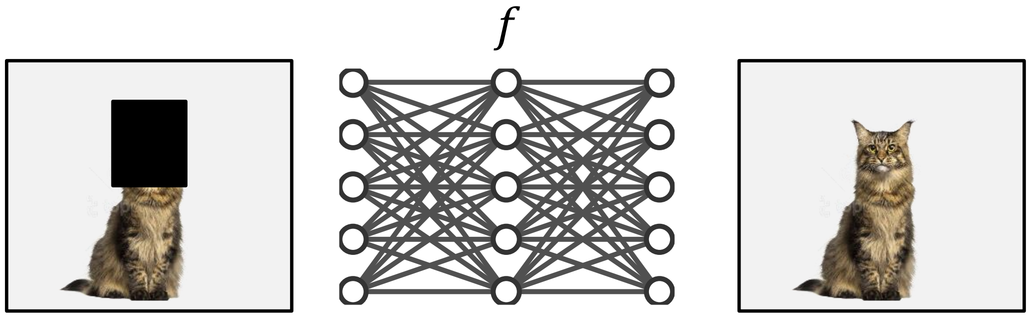
Ground-truth | Total variation (28.2 dB) | Deep network (**34.5 dB**)



x8 accelerated MRI [Zbontar et al., 2019]

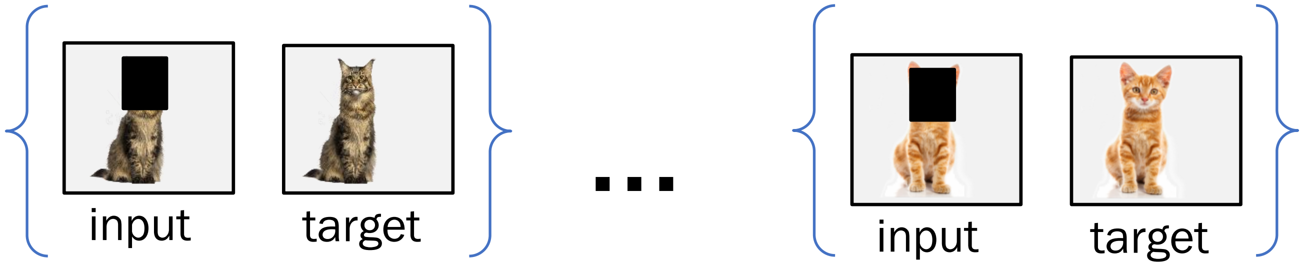
Learning approach

Idea: use training pairs of signals and measurements to directly learn the inversion function



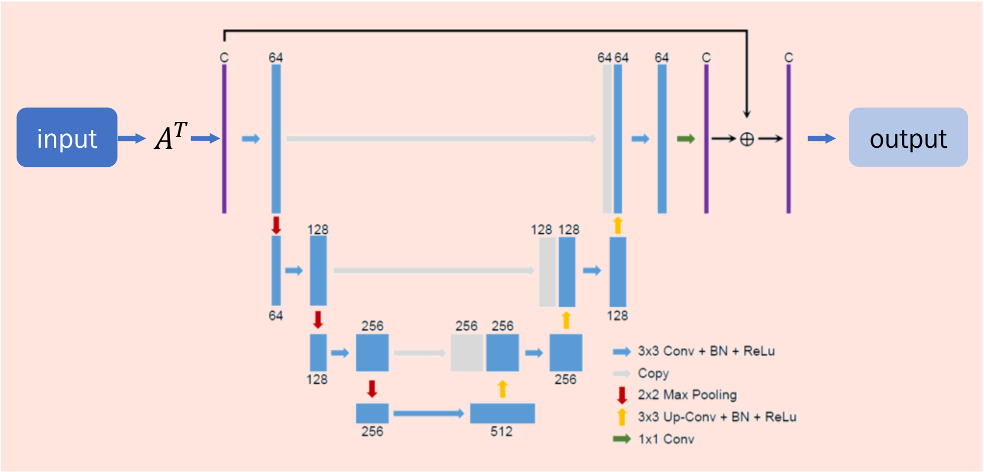
$$\underset{f}{\operatorname{argmin}}\, \mathbb{E}_{\boldsymbol{x},\boldsymbol{y}} \|\boldsymbol{x} - f(\boldsymbol{y})\|^2$$

supervised dataset

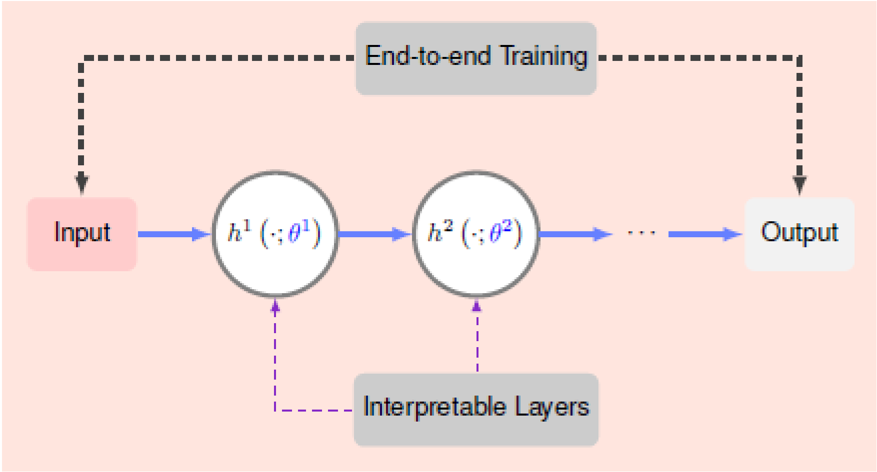


Learning approach

Many DNN architecture choices, e.g.



Back projected U-Net: $\hat{\boldsymbol{x}} = f(A^\top \boldsymbol{y})$, e.g. [Jin, 2017]



Unrolled networks: $\hat{\boldsymbol{x}} = f(\boldsymbol{y}, A)$, e.g. [Monga, 2020]

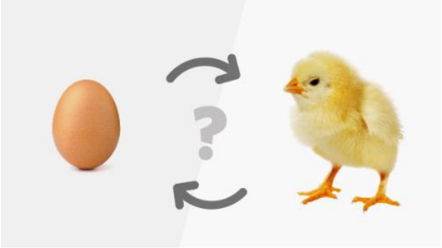
But also DnCNNs, DRUNet, SCUNet, DEQ, restormer, SwinIR, DiffPIR…

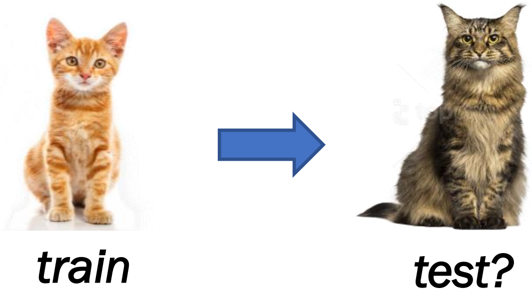
Here our focus will be on learning that is typically **architecture agnostic**

Learning approach

Main disadvantage: reference data can be expensive or impossible to get.

- Medical and scientific imaging
- Problems which we already ‘solved’
- Distribution shift



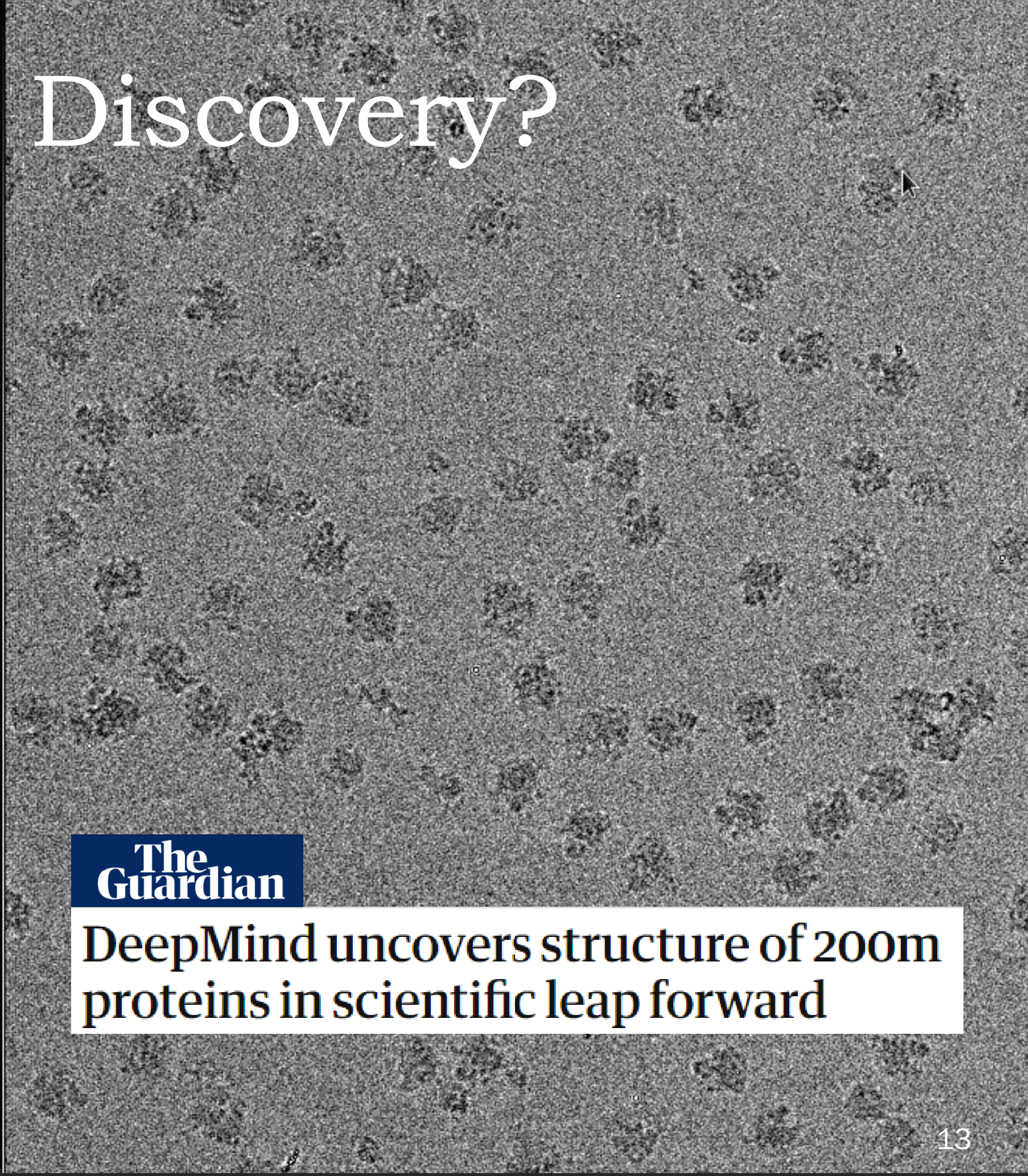


- **Raises the question**:
  Can AI be used for data-driven knowledge discovery in imaging?

AI for Knowledge Discovery?

The Guardian

Black hole picture captured for first time in space breakthrough



The Guardian

DeepMind uncovers structure of 200m proteins in scientific leap forward

Learning vs Inductive Bias

Inductive bias: all networks carry inductive bias

Deep Image Prior [Ulyanov 2018]

- Architecture *implicitly* encodes "preferred" images
- Minimise data consistency with early stopping

$$\underset{f}{\operatorname{argmin}} \|\boldsymbol{y} - Af(\boldsymbol{y})\|^2$$

Hard to categorise:
- Inductive bias?
- Learning non-local structure [Tachella 2021]?

Deep Image Prior [Ulyanov 2018]

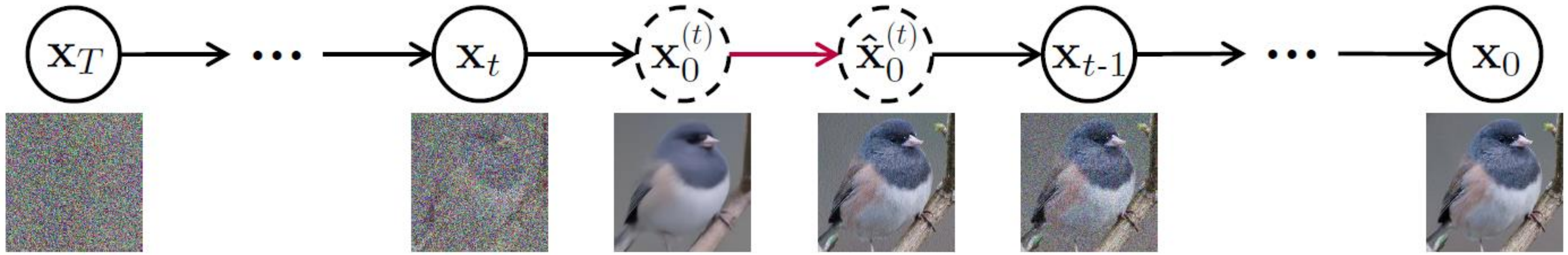
Learning vs Pre-Trained

Exploiting Deep Pre-Trained Denoisers: Exploit powerful pretrained DNN denoisers, $\mathrm{D}(\boldsymbol{u}, \sigma)$, e.g., [Zhu, 2023]

Plug and Play methods: e.g. PnP proximal gradient descent

$$\boldsymbol{u}_{k+1} = \mathrm{D}(\boldsymbol{x}_k - \gamma A^{\mathrm{T}}(A\boldsymbol{x}_k - \boldsymbol{y}), \sigma)$$

Conditional Diffusion models: use pre-trained denoiser in reverse SDE to attempt to sample conditional distribution



Generally pre-trained but can leverage self-supervised denoisers (see part III)

References

The full reference list for this tutorial can be found here:

https://tachella.github.io/projects/selfsuptutorial/